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Gradient Search Technique Applied to Dual-Optimal Design of a 3-Phase Core Type Transformer

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Abstract **- Most of the power and distribution transformers in power system are 3-phase core type transformers. They are extensively used- as such their cost is a sizable proportion of the total system cost. Therefore they are to be designed cost-optimally. The optimality is with reference to an objective function, generally in presence of constraints. The design variables are to be varied within their given bounds and the optimal solution is to be reached in finite number of steps, satisfying the given constraints. The paper presents a method for optimizing the design, in presence of constraints specified by the customer and the regulatory authorities, through gradient search technique. The objective function is a weighted combination of the cost of the transformer and the running losses which is not a mathematically framed but obtained from the transformer design sub-routine through computer programme. The objective of this paper is to find out above objective function without mathematically framed function which leads to inaccuracy for approximation due to magnetic saturation and nonlinearity present in the system.**

Most of the objective function is framed by taking few terms from a polynomial to avoid complicacy of the function, such as core loss expression obtained from curve with the variation of flux densities. The starting point has been chosen within the allowable parameter space- the steepest decent path has been followed for convergence. The step length has been judiciously chosen. The hyper surface has been found to be concave. As such no local minima problem is faced. The method is best as it its convergence is quickest.

Keywords: **Dual Optimal design, three Phase transformer, objective function, Computer subroutine**

I. INTRODUCTION

Previously engineering designs were made on empirical basis based on the accumulated experience of the designer. In a later period of time, analytical tools were developed on the basis of mathematical modeling. The engineers began to apply these tools to design problems. This approach involved many variables and constraints and multi-step calculations to reach a feasible solution. By this time fast-acting computers appeared. So, recourse to computer was made for solving design problems [1], [2], [3].

Four different approaches are there to solve a design problem can be identified viz.

Analytical design: In this method the designer chooses the values of the design variables by consulting charts and tables given in the text-books of design, backed up by his experience. Then he calculates the dimensions of the equipment and the performance variables. If a feasible or acceptable solution is not obtained by this procedure, the designer suitably modifies the values of design variables and restarts the process.

Synthetic design: The performance variables and some dimensions are prescribed in this method. In addition, there may be constraints. The program adjusts the design variables to reach the target as closely as possible without violating the constraints. The basis is empirical based on experience of the designer- it requires more skillful programming [4], [5], [6].

Optimal design: In this method an objective function is suitably chosen. This function has to be optimized (minimized or maximized) in presence of a set of equality and/or non-equality constraints. Through appropriate loops in the program, the optimal solution is reached without violating any of the constraints. This is accomplished either by classical methods (gradient search, Powell's method etc.) or by soft-computing techniques (e.g., GA, PSO etc.). A skillful programmer having high degree of mathematical knowledge can only adapt to this method. The optimal design may either be static or dynamic, constrained or unconstrained.

Standard design: This method is used in mass production to get best possible economy. The stampings, frame size etc. are all standardized for a series of standard mass production items. The manufacturing unit may itself standardize and produce the stampings or frames or it may purchase them straight-way from the market whichever is economically more advantageous.

The synthetic design is better than the analytic design, but people generally aim at the optimal design. Standard design methods are followed by the bulk manufacturers [1].

II.THE 3-PHASE CORE-TYPE TRANSFORMER

The 3-phase transformers generally employ core construction as it is more economic [7]. Which are either of the power type or distribution type. A power transformer operates almost at full load. So it is designed for maximum efficiency at or near its full load (for larger flux-density). A distribution transformer operates at an average load of 40% to 60% of its rated power, depending upon the load factor at the point of use. So they have to be designed for maximum efficiency at partial load (for lower flux-density). CRS-type cores are invariably used for all applications [8]. Aluminium is used as conductor in distribution transformers up to a certain size to achieve economy [9], [10].

Transformers in common application are oil-cooled. The core-coil structure is placed on a soft-bed within an oil-filled tank to reduce the noise level. The tank is provided with expansion tank, breather and protective devices like Buchholtz relay. Cooling tubes/radiators are added to increase the convection. For larger sizes, natural oil-cooling may have to be augmented by forced air-cooling or forced oil-cooling to keep the temperature within statutory limits.

III. REVIEW WORK

The present work is the development of simple and appropriate methodology for cost-optimal design of 3-phase core type transformer suitable for practical use, which can be compared with the performance of the transformer used in the industry. The design has to be made in presence of constraints. The objective of this paper as follows

1. Dual optimization, which is a weighted sum of the cost of production and cost of average annual losses.

Past and recent trend in Design optimization of transformers uses Soft computing tools which obtained from different literatures [11], [12], [13].

Therefore, objective functions are framed against idealizing assumptions due to non linearity and saturation present in the system which affect the accuracy of the solution. Both traditional and non-traditional methods of optimization are employed. Sometimes nontraditional (like GA & SA) are used more often to deal with the complex problem [14], [15], [16] and to get global minima point because it cover almost all the search area. The main difficulty related to this issue is objective and constraint functions are framed with approximations, which leads to the resulting dimensions of the motor different from the practical field may be some of the performances are closely matched (taken as constraints) due to upper and lower bounds.

1. Finally it is concluded from the literature survey is that; all the optimization is done on theoretical basis.
2. Performance variables like efficiency, no load current etc. when acting as constraints (restrictions) their values obviously matched with the industrial performance because of the bounds.
3. Dimensionally results are differing from the actual field data's due to assumptions and approximation imposed to give shape of the objective function.
4. Research work is done with an intention to give shape the objective function of an optimization problem to make it feasible mathematically, not to make it simple and accurate from industrial point of view.

IV. METHODOLOGY

Mathematical objective function will be replaced by Transformer Design subroutine, where cost itself a sub routine in order to avoid mathematical complicacy and assumptions (due to non linearity and saturation present in the system)that leads to get data as close as possible for industrial design.

1. Next step is to calculate the cost of production against variation of the design variables (without violating the constraints). This is obtained from the design subroutine.
2. Optimization procedure to be imposed through loops(proper programming required for different optimization procedure, which-ever is applicable)
3. Finally the values of the design variables are noted, corresponding to minimum cost, without violating the constraints. The performance variables and design parameters are calculated from the subroutine.

The method is not time-consuming now-a-days, due to tremendous speed of the computer- the optimal cost reaches within few seconds.

All the steps for design are covered up by a computer-subroutine- the subroutine itself can be used as the objective function. The method is simple and straight-forward and easily converges to an optimal solution. The constraints can be taken care of by fly-off from the subroutine as and when they are faced.

The optimal design will start from the choice of the objective function which is the cost of production. A better approach is to set the objective function as a weighted sum of the cost of production and the running cost. This is dual optimization, beneficial for the manufacturer as well as the consumer and then write up flow-chart and program.

V. CONSTRAINTS, OBJECTIVE FUNCTION AND DESIGN VARIABLES

The constraints appear due to rules imposed either by the regulatory authorities or by the customer. The efficiency should not fall below the limit specified by either of them. The voltage regulation should be kept within a maximum limit for distribution transformers- so their leakage reactance should be relatively low. The short-circuit current should be kept within a maximum limit for the power transformer- so their leakage reactance should be relatively high.

The allowable temperature rise depends on the type of transformer- whether oil-cooled or air-cooled. Dry type or bitumen-filled transformers are less hazardous but more expensive. In any case, the maximum allowable temperature rise must not be exceeded. The design variables should be chosen with a look to these points.

The key variables to be chosen to optimize a design problem depend on the objective function. While it is optimized to get minimum possible cost of production subject to usual design constraints, the iron loss and copper loss are kept at their maximum possible values to reduce the cost of production [17]. Accordingly, the flux density and the current density are kept at their maximum possible values without violating the design constraints. But if the customer's interest is to be secured, then the running cost towards lost energy units must also be included in the objective function [18]. Therefore, the flux density and the current density are also to be chosen as design variables to find the minimality conditions for the chosen objective function. In this paper, we have taken an objective function which covers the interest of both the manufacturer and the user. The objective function is a weighted sum of the cost of production and the cost of lost energy units.
In such a case, the following variables affect the objective function:

Design variables

a. The e.m.f. constant K (in eqn. $E_t = K\sqrt{S}$ where E_t = e.m.f. per turn, S =KVA rating).
b. The ratio of window height to window width: $R_w = H_w / W_w$
c. The maximum flux-density B_m
d. The maximum current-density, δ

VI. THE BOUNDS ON DESIGN VARIABLES

The design variables have been suggested, which affects the cost for a 3-phase core type transformer:
E.M.F constant, K: 0.45-6 for distribution transformer 0.6-0.7 for power transformer (with copper conductors, somewhat smaller for Aluminium)
Window height/width R_w: 2.0-3.0 for power transformer 3.0-4.0 for distribution transformer.

The following choice of materials has been recommended:

Core material: CRNOS for smaller ratings, CRGOS for larger ratings.
Conductor materials: Aluminium for smaller ratings, Copper for larger ratings

After choosing the conductor and the core material judiciously, our task is to choose such values of $K, R_w, B_m \,\&\, \delta$ which gives minimality of the objective function without violating the design constraints. After choosing the core and the conductor, the minimality is being sought against these four variables [19].

At first, a suitable starting point is chosen within the allowable zone of design variables and the objective function is evaluated at this point. Then the variables are given small increments and the corresponding values of the objective functions are again evaluated. This enables us to get: $\partial C / \partial K$, $\partial C / \partial R_w$, $\partial C / \partial B_m$ and $\partial C / \partial \delta$. Then a step towards the direction of steepest decent is made in the 4-dimensional hyper-surface. The step-length is to be chosen judiciously- not too small to avoid burden of computation, not too large as to skip the minimal point.

VII. ALGORITHM

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Step 1: Input specifications of the transformer
Step 2: Input user-specified data for design variables: no. of core steps, $N_{st}$ etc.; Step lengths: $\lambda_k, \lambda_{rw}, \lambda_{bm}, \lambda_\delta$, minimum cost $C_{\min}$ (a large number)
Step 3: Choose copper as conductor material, CRS as core material. Input associated data.
Step 4: Initialize:
    $K \leftarrow 0.45;\ R_w (= H_w / W_w) \leftarrow 3.0;\ B_m \leftarrow 1.5\ Tesla;\ \delta \leftarrow 2.8\ A/mm^2$
Step 5: Go to transformer design sub-routine: Find the objective functions $C$ and evaluate the performance variables: If there be any constraint violation go to step 2
Step 6: If $C < C_{\min}$ then $C_{\min} \leftarrow C$ else go to step 16
Step 7: $K \leftarrow 1.01K$ ; Go to transformer design sub-routine: Find cost $CK$
Step 8: $K \leftarrow K / 1.01$
Step 9: $\partial C / \partial K \leftarrow (CK - C) / (0.01K)$
Step 10: $R_w \leftarrow 1.01R_w$ ; Go to transformer design sub-routine: Find cost $CR$
Step 11: Set $R_w \leftarrow R_w / 1.01$
Step 12: $\partial C / \partial R_w \leftarrow (CR - C) / (0.01R_w)$
Step 13 $B_m \leftarrow 1.01B_m$ ; Go to transformer design sub-routine: Find cost $CB_m$
Step 14: Set $B_m \leftarrow B_m / 1.01$
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Step 15: $\partial C / \partial B_m \leftarrow (CB_m - C)/(0.01B_m)$

Step 16: $\delta \leftarrow 1.01\delta$; Go to transformer design subroutine: Find cost $C\delta$

Step 17: Set $\delta \leftarrow \delta / 1.01$

Step 18: $\partial C / \partial \delta \leftarrow (C\delta - C)/(0.01\delta)$

Step 19: $K \leftarrow K - (\partial C / \partial K)\lambda_k$

Step 20: $R_w \leftarrow R_w - (\partial C / \partial R_w)\lambda_{rw}$

Step 21: $B_m \leftarrow B_m - (\partial C / \partial B_m)\lambda_{bm}$

Step 22: $\delta \leftarrow \delta - (\partial C / \partial \delta)\lambda_\delta$

Step 23: Go to step 5

Step 24: Go to transformer design subroutine with the current values of $K, R_w, B_m \& \delta$.

Find dimensions, cost, and performance variables.

Step 25: Print out results

Step 26: Stop

Step 27: End

Constraints: efficiency $\geq$ 97%; voltage regulation $\leq$ 5%; no load current $\leq$ 1.0%

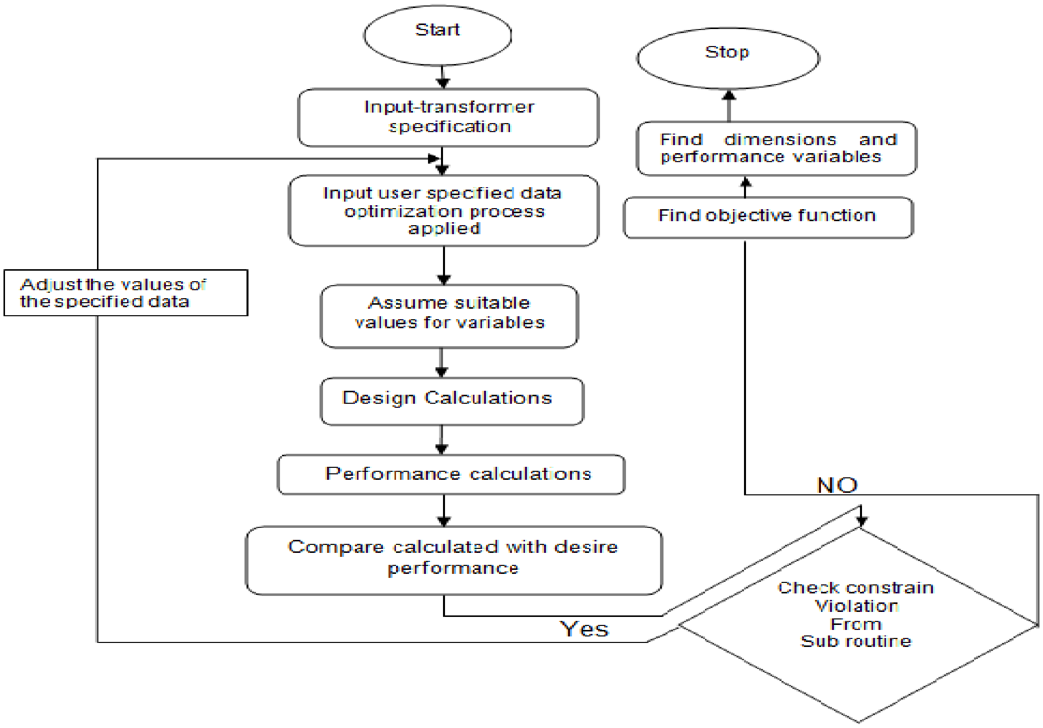

Fig.1 Flowchart

TABLE 1 THE CONVERGENCE TABLE

K(e.m.f Constant)	**R_W(Window Height /Width Ratio)**	**B_m(Flux Density)**	**∂(Current Density)**
0.485025	4.08125	1.39707	2.777375
0.497475	3.94875	1.4462	2.853575
0.497475	3.94875	1.4462	2.853575

VIII.CASE STUDY

The cost optimal design of a 3-phase oil-filled distribution transformer of 1000 KVA, 11000/433 V and DY-connected. CRGOS has been chosen as the core material and Copper as a conductor. Dual optimization has been made with a look to the interest of the customer and the manufacturer. The objective function is a weighted combination of the cost of production and the annual energy loss.

The objective function minimizes for:
kVA rating of the machine= 1000
EMF-constant, K= 0.497475
Window height/width ratio, R_w= 3.94875
Flux-density, Bm, Tesla= 1.4462
Current density, A/sq.mm, δ= 2.853575
Against specific costs:
Copper: Rs. 530 /Kg
Core iron: Rs. 170 /Kg
Tank wall: Rs. 90 /Kg
Transform roll: Rs. 97 /Kg

TABLE II THE DESIGN DETAILS OF THE OPTIMAL MACHINE

1. Nominal frequency in Hz.= 50
2. Primary Connection: DELTA
3. Secondary Connection: STAR
4. Conductor material: COPPER
5. No. of taps= 5
6. The EMF- constant= 0.497475
7. No of nominal turns of the primary= 699
8. No of addl. turns of the primary for tapping= 34
9. Total no of turns of the primary= 733
10. No of nominal turns of the secondary= 16
11. Current in Primary/ Secondary, A: 30.30303/ 1333.372
12. Chosen current density in A/sq.mm= 2.853575
13. Cross section of primary/ secondary, sq.mm: 10.61932 /467.2638
14. Net area of core iron, sq.mm= .0490483
15. Stacking factor= 0.92
16. Gross area of core iron, sq.mm= 5.331337E-02
17. Diameter of the core circle= 0.2842144
18. Length of the core sides in mm: 257 : 201 : 121
19. Area of the window, sq.mm= 0.1216544
20. Window height/width: 0.6930966 /0.175523
21. Distance between core centers= 0.4327371
22. Width/height of yoke: 0.25721 /0.20727
23. Total length of core,m= 1.176689
24. Total height of core, m= 1.107642
25. Iron loss in W= 2996.226
26. % Iron loss= 0.2996226
27. Mean length of turn of primary/ secondary: 1.003173 / 1.278884
28. Resistance of Primary/ Secondary, ohm: 1.385752 9.124527E-04
29. Copper loss in W= 8684.199
30. % Copper loss = 0.8684199
31. Total % loss = 1.168043
32. Efficiency at full load & .8 p.f= 0.9856096
33. Maximum efficiency of 0 .9899011 occurs at a % load of 58.73844
34. The magnetizing current in %= 0.5518291
35. The core loss current in %= 0.2996226
36. The no load current in %= 0.6279244
37. The % leakage reactance= 3.448458
38. The % voltage regulation at rated power & power factor = 2.763811
39. The tank length, width, height: .573 * 1.452 * 1.258
40. The no. of tubes (50 mm dia.) required= 179
41. The weight, Kg/ cost of tank: 467.4966 /Rs. 42075 /-
42. The volume, liter; cost of oil: 1.046025 /Rs. 101464 /-
43. Volume of iron, m^3= 0.2174148
44. Weight of iron, Kg= 1706.706
45. Cost of iron, Rs. 290140 /-
46. Volume of copper,m^3= 0.0519008
47. Weight of copper, Kg= 461.9171
48. Cost of copper, Rs. = 244816 /-
49. Direct cost allowing 25 % labour charge= Rs. 848119 /-
50. Selling cost allowing 35 % overhead = Rs. 1144961 /-
51. Annual cost of lost energy at Rs. 5 /BOT =Rs. 328075 /-
52. Cost function (cost of production+ 7 years of energy loss=Rs.3441487 /-

TABLE III COMPARISON OF PERFORMANCE

Performance of Optimal Machine	**Performance of Field Machine**
1. Iron loss in W= 2996.226	1. Iron loss in W= 3012.234
2. Copper loss in W= 8684.199	2. Copper loss in W= 8762.095
3. Total % loss = 1.168043	3. Total % loss = 1.328013
4. Efficiency at full load & .8 P.f= 0.9856096	4. Efficiency at full load & .8 P.f = 0.9616292
5. The no load current in %= 0.6279244	5. The no load current in %= 0.8219234
6. The % voltage regulation at rated power & P.f= 2.763811	6. The % voltage regulation at rated power & Power factor = 2.83251
7. Weight of iron, Kg= 1706.706	7. Weight of iron, Kg= 1790
8. Cost of iron, Rs. 290140 /-	8. Cost of iron, Rs. 299100 /-
9. Weight of copper, Kg= 461.9171	9. Weight of copper, Kg= 465.234
10. Cost of copper, Rs. = 244816 /-	10. Cost of copper, Rs. = 260910 /-

IX. CONCLUSION

The cost of transformers in a power system is an appreciable fraction of the total cost. So they must be designed cost-effectively. The design methodology is inadequate in the sense that it fails to give a cost-optimal solution and even fail to give a feasible solution provided the design variables are freely chosen. Also there is no feedback from the values of performance variables in the process. There is as such no check against their unsatisfactory values.

There are several methods for reaching the optimal solution in presence of design constraints. Though such methods e.g. Monte Carlo random walk, Powell's method, Gradient search technique etc. are complex, they are very effective in the sense that they give the most economic design. In this paper, gradient search technique has been used. However, no attempt has been taken to develop an expression for the objective function. The total cost of the transformer computed by a transformer design subroutine has been taken as the objective function. The gradients are being calculated at each experimental point and then a step is being taken towards steepest decent. The starting point and the step length have been judiciously chosen. The local/global minima problem is absent in this case as the surface has been found to be concave, which is already, tested by using other programs.

The case study has been made on a 1000 KVA, 11000/433 V, oil-filled, DY-connected, distribution transformer. CRGOS has been chosen as the core material and copper has been used as the conductor material [20]. The cost-optimal solution has been reached by gradient search technique finally it is compared with the actual data.

REFERENCES

[1] M. Ramamoorty, "Computer-aided design of electrical equipment", *Affiliated East-West Press Pvt. Ltd.*, 1987, ISBN: 81-85095-57-4

[2] M.G. Say, *Performance and design of A.C. machines*, ELBS.

[3] A.K. Sawhney, "*A course in electrical machine design*", Dhanpat Rai and Sons, 2003.

[4] S.B. Williams, P.A. Abetti and E.F. Mangnusson, "Application of digital computers to transformer design", *AIEEE trans.*, Vol-75, pt-III, pp. 728-35, 1956.

[5] W.A. Sharpley and J.B. Oldfield, "The digital computer applied to the design of large power transformers", *Proc. IEEE,* Vol-105, pt-A, pp. 112-25, 1958.

[6] S.B. Williams, P.A. Abetti and H.J. Mason, "Complete design of power transformer with a large size digital transformer", *AIEEE trans*., Vol. 77, pt-III, pp.1282-91, 1959.

[7] Dymkov, *Transformer design*, MIR publications.

[8] A.Shanmugasundaram, G. Gangadharan and R. Palani *Electrical machine design data book*, Wiley Eastern Ltd. ISBN 0 85226 813

[9] O. W. Andersen "Optimized design of electric power equipment" *IEEE Computer Applications in Power* Vol.4, No.1,pp. 11,1991.

[10] N. D. Doulamis and A. D.Doulamis, "Spotlight on Modern Transformer Design" *International Conference on Systems, Man and Cybernetics, Proc. of IEEE, 5*, 2002.

[11] P. S. Georgilakis, M. A. Tsili and A. T. Souflaris "A novel iron loss reduction technique for distribution transformers based on a combined Genetic Algorithm-neural network approach" *Journal of materials processing Technology*,Vol. 181, No.1,pp 260-266 , 2007.

[12] S. Elia, G. Fabbri, E. Nistico and E. Santini " Design of cast-resin Distribution Transformers by means of Genetic Algorithms" *International Symposium on Power Electronics, Electrical drives, automation and motion, speedam, 1473*,2006

[13] N. Tutkun and A. Moses "Design optimisation of a typical strip-wound toroidal core using Genetic Algorithms" *Journal on Magnetism and Magnetic materials*, Vol. 277(1-2)216, 2004.

[14] R.A Jabr "Application of Geometric Programming to Transformer Design" *IEEE Transactions on Magnetics* Vol. 41, No.11, pp.4261-4169, 2005.

[15] E.I Amoiralis, P.S Georgilakis and A.T Gioulekas "An artificial neural network for the selection of winding material in power transformers" *Advances in Artificial Intelligence, 4th Hellenic Conference on AI*, pp.465-468, 2006.

[16] K. Deb, "*Optimization for engineering design*", PHI, 2010. ISBN 978-81-203-0943

[17] O.W. Anderson, "Optimum design of electrical machines," *IEEE Trans. (PAS),* Vol-86, pp. 707-11, 1967.

[18] S.S. Rao, "*Engineering optimization- theory and practice*", New Age Int. ; ISBN 978-81-224-2723-3

[19] N.S. Kambo, *Mathematical programming techniques*, revised edition, 1991, 1984, *Affiliated East-West Press Pvt. Ltd.* New Delhi – 110 001,ISBN 81-85336-47-4.

[20] C. J. Wu and F. C. Lee "Minimum weight EI core and pot core inductor and transformer designs" , *IEEE Transactions on Magnetics* Vol.16, No.5, pp. 755,1980.